

LOGARITHM & QUADRATIC EQUATION

Factors Theorem :- If $P(x)$ be the Poly of $\deg \geq 1$ & a is a Real No. ($a \in \mathbb{R}$) then $(x-a)$ will be factor of $P(x)$ if $P(a) = 0$.

Q. If $P(x)$ is divided by $(x-1)$ then $\text{Rem} = 5$ & when divided by $(x+2)$

$\text{Rem} = -1$ find Rem of $P(x)$ is divided by $(x-1)(x+2)$?

$$P(1) = 5 \quad \& \quad P(x) = (x-1)Q_1(x) + 5$$

$$P(-2) = -1 \quad \& \quad P(x) = (x+2)Q_2(x) + -1$$

$$P(x) = (x-1)(x+2)Q_3(x) + \boxed{ax+b}^{\text{Rem}}$$

$$P(1) = 0 + a + b \Rightarrow 5 = a + b$$

$$P(-2) = 0 - 2a + b \Rightarrow -1 = -2a + b$$

$$a + b = 5$$

$$-2a + b = -1$$

$$3a = 6 \Rightarrow a = 2, b = 3$$

$$\text{Rem} = ax + b = 2x + 3$$

LOGARITHM & QUADRATIC EQUATION

Q If $P(x)$ is divided by $(x-1)$ then Rem = 2 & if it is divided by $(x-2)$ then Rem = 1
 find Rem when $P(x)$ is divided by $(x-1)(x-2)$?

$$P(1) = 2 \quad \& \quad P(x) = (x-1)Q_1(x) + 2$$

$$P(2) = 1 \quad \& \quad P(x) = (x-2)Q_2(x) + 1$$

$$P(x) = \underbrace{(x-1)(x-2)}_{Q \in \mathbb{Q}^n} Q_3(x) + \boxed{ax+b}$$

यहाँ $\deg$ 1 का $\deg$.

$$x=1 \rightarrow P(1) = 0 + a+b \Rightarrow a+b = 2$$

$$x=2 \rightarrow P(2) = 0 + 2a+b \Rightarrow 2a+b = 1$$

$$-a = 1 \Rightarrow a = -1, b = 3.$$

$$\text{Rem} = ax+b = -x+3.$$

LOGARITHM & QUADRATIC EQUATION

Q If Poly $P(x)$ is divided by $(x-1)$ then $\text{Rem} = 1$, if it is divided by $(x-4)$ $\text{Rem} = 10$
 find Rem if $P(x)$ is divided by $(x-1)(x-4)$?

$$P(1) = 1 \text{ \& } P(x) = (x-1)Q_1(x) + 1$$

$$P(4) = 10 \text{ \& } P(x) = (x-4)Q_2(x) + 10$$

$$P(x) = (x-1)(x-4)Q_3(x) + \boxed{ax+b}$$

$$\begin{aligned} \text{Rem} &= ax+b \\ &= 3x-2 \end{aligned}$$

$$\left. \begin{aligned} P(1) &= a+b \Rightarrow a+b=1 \\ P(4) &= 4a+b \Rightarrow 4a+b=10 \end{aligned} \right\} \begin{aligned} a+b &= 1 \\ 4a+b &= 10 \end{aligned}$$

$$\begin{array}{r} \underline{4a+b=10} \\ -a+b=1 \\ \hline -3a=-9 \end{array} \Rightarrow \boxed{a=3}, b=-2$$

LOGARITHM & QUADRATIC EQUATION

Mains
problem

Find Rem of $x^{135} + x^{125} - x^{115} + x^5 + 1$ is divided by $x^3 - x$?

$$\begin{aligned} \text{Rem} &= ax^2 + bx + c \\ &= 0 \cdot x + 2x + 1 \\ &= 2x + 1 \end{aligned}$$

$$\begin{aligned} P(x) &= x^{135} + x^{125} - x^{115} + x^5 + 1 \\ P(0) &= 1, P(1) = 1 + 1 - 1 + 1 + 1 = 3 \\ P(-1) &= -1 + (-1) - (-1) + (-1) + 1 = -1 \end{aligned} \quad \begin{array}{l} x^3 - x \\ (x)(x-1)(x+1) \\ \downarrow \quad \quad \quad \downarrow \quad \quad \downarrow \\ a=0, 1, -1 \end{array}$$

$$P(x) = \underbrace{(x^3 - x)}_{\text{Cubic}} Q(x) + \boxed{ax^2 + bx + c}_{\text{Rem.}}$$

$$P(0) = 0 + c \Rightarrow c = 1$$

$$P(1) = 0 + a + b + c \Rightarrow a + b + 1 = 3 \Rightarrow \boxed{a + b = 2}$$

$$P(-1) = 0 + a - b + c \Rightarrow a - b + 1 = -1 \Rightarrow \boxed{a - b = -2}$$

$$2a = 0 \Rightarrow a = 0, b = 2$$

LOGARITHM & QUADRATIC EQUATION

Ratio & Proportion.

1) Ratio:- If a & b are of same type then Ratio is rep. as $a:b$ or $\frac{a}{b}$.

2) $a:b$ = Ratio $a^2:b^2$ = Duplicate Ratio $a^3:b^3$ = Triplicate Ratio	$\sqrt{a}:\sqrt{b}$ = Sub duplicate Ratio $a^{1/3}:b^{1/3}$ = Sub Triplicate Ratio
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(3) Compounded Ratio:- for 3 Ratios $a:b$, $c:d$, $e:f$

Compounded $\rightarrow \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$

LOGARITHM & QUADRATIC EQUATION

(D) If a, b & c, d are in Proportion

then it means $\Rightarrow a : b :: c : d$

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = c \times b$$

2) Prod of extreme = Prod of mean

(E) Continued Proportion :- If a, b, c are in Continued Proportion.

$$a : b :: b : c$$

$$\frac{a}{b} = \frac{b}{c} \Rightarrow \boxed{ac = b^2}$$

If a, b, c, d are in Continued Proportion.

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

LOGARITHM & QUADRATIC EQUATION

(F) Compendendo

$$\text{If } \boxed{\frac{a}{b} = \frac{c}{d}} \Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1$$

$$\Rightarrow \boxed{\frac{a+b}{b} = \frac{c+d}{d}}$$

(G) Dividendo

$$\boxed{\frac{a}{b} = \frac{c}{d}} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1$$

$$\Rightarrow \boxed{\frac{a-b}{b} = \frac{c-d}{d}}$$

DROPPER

12+11
8 Month.

H) Compendendo & Dividendo $\Rightarrow$ Niche wala add
Niche wala Sub.

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \boxed{\frac{a+b}{a-b} = \frac{c+d}{c-d}}$$

LOGARITHM & QUADRATIC EQUATION

Q $\frac{x}{y} = \frac{3}{4}$ Use C&D?

$$\frac{x+y}{x-y} = \frac{3+4}{3-4}$$

Q. $\frac{\cancel{\cos \theta} + \cancel{\sin \theta}}{\cancel{\cos \theta} - \cancel{\sin \theta}} = \frac{\cancel{x} + \cancel{y}}{\cancel{x} - \cancel{y}}$ find $\frac{x}{y} = ?$

Shakl hi (C&D) Jaisi hai!!

Trick: $\boxed{\frac{\cos \theta}{\sin \theta} = \frac{x}{y}}$ Proper Method.

$$\frac{2\cos \theta}{2\sin \theta} = \frac{2x}{2y}$$

$$\frac{(\cancel{\cos \theta} + \cancel{\sin \theta}) + (\cancel{\cos \theta} - \cancel{\sin \theta})}{(\cancel{\cos \theta} + \cancel{\sin \theta}) - (\cancel{\cos \theta} - \cancel{\sin \theta})} = \frac{(\cancel{x} + \cancel{y}) + (\cancel{x} - \cancel{y})}{(\cancel{x} + \cancel{y}) - (\cancel{x} - \cancel{y})}$$

LOGARITHM & QUADRATIC EQUATION

(H) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then

$$\text{Ex: } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$$

$$\text{Ex } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a-c+e}{b-d+f}$$

$$\text{Ex: } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{2a-3c+5e}{2b-3d+5f}$$

$$\text{Ex: } \frac{x}{y} = \frac{m}{n} \Rightarrow \boxed{\frac{x}{y} = \frac{m}{n} = \frac{3x-4m}{3y-4n}}$$

(I) K method

Proof: Let $\frac{x}{y} = \frac{m}{n} = k$

$$\Rightarrow x = ky, m = nk$$

$$\frac{3x-4m}{3y-4n} = \frac{3ky-4nk}{3y-4n}$$

$$\frac{k(3y-4n)}{(3y-4n)} = k$$

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Ex: $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{ax+cy+ez}{bx+dy+fz} \mid \frac{m}{n} = \frac{f}{g} = \frac{(3m^5+4f^5)^{1/5}}{(3n^5+4g^5)^{1/5}}$

Adn $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{(xa^n + yb^n + ze^n)^{1/n}}{(xb^n + yd^n + zf^n)^{1/n}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$
 $a = bk, c = dk, e = fk$
 $a, b, c = D.R., l, m, n = D.C.$

$$\frac{(xb^n k^n + yd^n k^n + zf^n k^n)^{1/n}}{(xb^n + yd^n + zf^n)^{1/n}} = \frac{k(xb^n + yd^n + zf^n)^{1/n}}{(xb^n + yd^n + zf^n)^{1/n}} = k$$

LOGARITHM & QUADRATIC EQUATION

$$a, b, c = DR$$

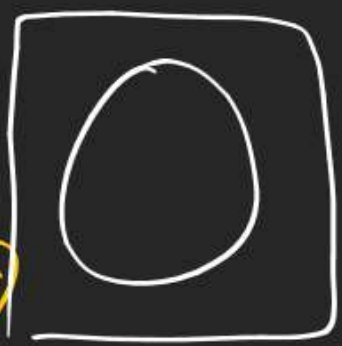
$$l, m, n = DC$$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{(l^2 + m^2 + n^2)^{1/2}}{(a^2 + b^2 + c^2)^{1/2}} = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

LOGARITHM & QUADRATIC EQUATION

Q If $x = \frac{\sqrt{2a+3b} + \sqrt{2a-3b}}{\sqrt{2a+3b} - \sqrt{2a-3b}}$ (8D Jaisy Shkl) then find $3bx^2 - 4ax + 3b = ?$



$\frac{x+1}{x-1} = \frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}$ (Sqr.) $\frac{x^2+1+2x}{x^2+1-2x} = \frac{2a+3b}{2a-3b}$ (8D Jaisi Shkl)

$\frac{2(x^2+1)}{2x} = \frac{2a}{3b}$ (8D) $\frac{(x^2+1+2x) + (x^2+1-2x)}{(x^2+1+2x) - (x^2+1-2x)} = \frac{2a}{3b}$

$\Rightarrow 3bx^2 - 4ax + 3b = 0$

LOGARITHM & QUADRATIC EQUATION

Q If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$ then find $x:y:z$?

Let $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4} = k \Rightarrow$

$$\begin{aligned} x+y &= 2k \\ y+z &= 3k \\ z+x &= 4k \end{aligned}$$

$$2(x+y+z) = 9k$$

$$\begin{aligned} x:y:z &= \frac{3k}{2} : \frac{k}{2} : \frac{5k}{2} \\ &= 3:1:5 \end{aligned}$$

$$\begin{array}{r} x+y+z = \frac{9k}{2} \\ x+y = 2k \\ \hline z = \frac{5k}{2} \\ x+y+z = \frac{9k}{2} \\ y+z = 3k \\ \hline x = \frac{3k}{2} \\ x+y+z = \frac{9k}{2} \\ z+x = 4k \\ \hline y = \frac{k}{2} \end{array}$$

LOGARITHM & QUADRATIC EQUATION

Q If we have $\frac{a}{b} = \frac{c}{d}$ then $\frac{a^2+b^2}{c^2+d^2}$ is

Not
Imp

A) $\frac{1}{2}$ B) $\frac{a+b}{c+d}$

C) $\frac{a-b}{c-d}$

(D) $\frac{ab}{cd}$

let $\frac{a}{b} = \frac{c}{d} = k$

$\frac{a}{c} = \frac{b}{d}$

$\left. \begin{array}{l} a = bk \\ c = dk \end{array} \right\}$

$$\frac{a^2+b^2}{c^2+d^2} = \frac{b^2k^2+b^2}{d^2k^2+d^2} = \frac{b^2(\cancel{k^2}+1)}{d^2(\cancel{k^2}+1)} = \frac{b^2}{d^2}$$

$$\frac{b^2}{d^2} = \frac{\cancel{b} \times \cancel{b}}{\cancel{d} \times \cancel{d}} = \frac{b}{d} \times \frac{a}{c} = \frac{ab}{cd}$$

LOGARITHM & QUADRATIC EQUATION

LOGARITHMS.

John Napier

36 (N)

↓ Exponential form

$$6^2 = 36$$

Antilog form

Antilog of x to Base a
 $=$ Exp of x to base a .

Q Antilog of $\frac{3}{4}$ of Base 16 = ?

$$= 16^{3/4} = (2^4)^{3/4} = 2^3 = 8$$

log form

$$\log_6 36 = 2$$

$$\log_9 81 = 2 \quad 9^{-1} = \frac{1}{9}$$

$$\log_9 9 = 1$$

$$\log_9 \frac{1}{9} = -1$$